

REGULAR TALKS : LIST OF ABSTRACTS

JULIEN CASSAIGNE (IML, MARSEILLE, FRANCE)

Words with complexity $p(n) = n + o(n)$.

Our aim is to describe, using substitutions, infinite words with factor complexity just above that of Sturmian words, namely such that $p(n)/n$ tends to 1. We will first review the results by Ali Aberkane using Rauzy graphs, where the words are assumed to be recurrent. Then we will discuss the general case (work in progress with Mitali Thatte from IISER Pune).

AMEYA DAIGAVANE (IITG, GUWAHATI, INDIA)

2-uniform words: cycle graphs, and an algorithm to verify specific word-representations of graphs.

For an arbitrary word w on an alphabet, we can define the alternating symbol graph, $G(w)$, as the graph in which the edge (a, b) is in E iff the letters a and b alternate in the word w . A graph $G = (V, E)$ is said to be word-representable if $G = G(w)$ for some word w on V . The general problem of checking whether a graph is word-representable has been shown to be NP-complete [2]. However, checking whether a given graph is a 2-uniform word-representable (each letter occurring exactly twice in the word), i.e., a cycle graph, has an $O(V^2)$ -time algorithm, described by Spinrad [3]. Related to this problem, we propose a novel $O(V \log V + E)$ -time algorithm, implementing Fenwick trees [4] to check whether a specific 2-uniform word w is a word-representative of a given graph $G = (V, E)$, i.e., if $G = G(w)$. We also investigate a specific class of circle graphs – the cycle graphs – and completely characterize the 2-uniform words representing them. In [1], one of the word-representatives w_n of the cycle graph C_n was given. We prove [5] that every 2-uniform word-representative of the cycle graphs can be obtained by reflections, or cyclic shifts followed by reflections of w_n , proving that their number is precisely $4n$.

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PAMELA FLEISCHMANN (LOUGHBOROUGH UNIVERSITY, UK)

Local Patterns.

A pattern is a word consisting of constants from an alphabet Σ of terminal symbols and variables from a set X . Given a pattern α , the decision-problem whether a given word w may be obtained by substituting the variables in α for words over Σ is called the matching problem. While this problem is, in general, NP-complete, several classes of patterns for which it can be efficiently solved are already known. We present two new classes of patterns, called k -local, and nested, and show that the respective matching problems can be solved efficiently for any fixed k . Moreover, we also show that testing membership of these classes can be done in polynomial time, and finally obtain that computing descriptive patterns for these classes as well.

VINCENT JOST (GSCOP, GRENOBLE, FRANCE)

Regular words: some conjectures to go beyond Fraenkel.

What is the most "regular" word for a prescribed density vector? if the occurrences of each letter form a congruence sequence, this is (defined to be) ideal, and similarly, if the letters define balanced sequences. But not all density vectors are balanceable. For instance, $(abacab)^*$ and $(abacba)^*$ are the two natural solution candidates for the vector $(1/2, 1/3, 1/6)$, so this vector is not balanceable. We discuss various open questions regarding this domain: counting the number of "regular" words, (the computational complexity of) recognizing regularizable density vectors, and defining suitable optimization problems for vectors that are not balanceable. We conjecture, among many other things, that a word is (strictly) billiard, if and only if, the word induced by it on every pair of letters of its alphabet is balanced. This conjecture arose after observing that a word can be (easily) reconstructed, knowing only its projection on every pair. We mention a recent result of J. O. Shallit, regarding the enumeration of certain regular words, making use of the Möbius function – a result that we observed experimentally 4 years ago with a student. The talk will be given so as to facilitate discussions with other researchers interested in these questions, as well as to introduce others to this topic. It will be informal – apart from precise definitions – non-technical, based on examples and drawings, and of course, many open questions and new conjectures will be provided.

MANASI KULKARNI (WATERLOO, CANADA, MADRAS, INDIA)

Relative Watson-Crick Primitivity of Words.

We introduce the concept of relative Watson-Crick primitivity of words and its generalization, the relative θ -primitivity of words, where θ is a morphic or an antimorphic involution. Similar to relatively prime integers that do not share any common factors, we call two words u and v relatively θ -primitive if they do not share a common θ -primitive root. We study some combinatorial properties of relatively θ -primitive words, as well as establish relations between each of the two words u and v and the result of some binary word operations between u and v , from this perspective.

REVEKKA KYRIAKOGLU (LABORATOIRE D'INFORMATIQUE GASPARD-MONGE,
MARNE-LA-VALLÉE FRANCE)

Recognizable morphisms.

The concept of recognizability of morphisms originates in the paper of Martin [2] under the term of "determination". The term "recognizability" was first used by Host in his paper in Ergodic Theory of Dynamical Systems [1]. The main theorem concerning recognizability was established by Mossé [3], who proved that any primitive non-periodic morphism is recognizable, provided one uses a two-sided notion of recognizability, instead of the one-sided notion used previously. The notion of recognizability came into full bloom after the interest shown by many scientists due to its various theoretical applications in several topics, from combinatorics on words to symbolic dynamics. A similar notion is "circularity", introduced by Cassaigne and later studied by Starosta. The two terms are often, but not always, used as synonyms. This lack of consistency in the literature could lead to confusion. To the best of the author's knowledge, there is not yet any study that collects these definitions and proves their equivalence or indicates their differences. In the current study, we provide a solid approach using a coherent definition of recognizability and circularity, independent of the multiple classical definitions. We prove that circularity is equivalent to recognizability under appropriate hypotheses. We also characterize one-sided recognizability.

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ZSUZSANNA LIPTÁK (UNIV. VERONA, ITALY)

On the Parikh-de-Bruijn grid.

We introduce the Parikh-de-Bruijn grid, a graph whose vertices are fixed-order Parikh vectors, and whose edges are given by a simple shift operation. This graph gives structural insight into the nature of sets of Parikh vectors as well as that of the Parikh set of a given string. We show its utility by proving some results on Parikh-de-Bruijn strings, the abelian analog of de-Bruijn sequences.

OLGA PARSHINA (ICJ LYON, SOBOLEV, RUSSIA)

Open and closed factors of Arnoux-Rauzy words.

We study the distribution of closed and open factors in Arnoux-Rauzy words. The notion of closed word was introduced by Gabriele Fici in 2011 to classify trapezoidal words. A word u is closed if and only if it is empty or has a proper factor occurring exactly twice in u , as a prefix and as a suffix of u . Otherwise u is open. Closed words are also known as periodic-like words and complete first returns. Given an Arnoux-Rauzy word w and a positive integer n , consider two functions counting the number of closed and open factors of length n in the word w , respectively. We provide explicit formulae to compute the values of these functions and study their behavior.

JOSÉ MANUEL RODRIGUEZ CABALLERO (UQAM, MONTREAL, CANADA)

Une approche langagière à la théorie élémentaire des nombres.

Hausel, Letellier and Rodriguez-Villegas (2013) et, indépendamment, Kassel et Reutenauer (2015) ont calculé les coefficients du E-polynôme du schéma de Hilbert de n points dans un tore algébrique. Ces coefficients, contenant de l'information arithmétique sur n , peuvent être codés par une expression bien parenthésée, ce qui permet de dégager de théorèmes arithmétiques en utilisant de rapports parmi des expressions bien parenthésées. Dans cet exposé, je me propose de montrer quelques exemples de théorèmes arithmétiques démontrés par cette méthode.

MATTHIEU ROSENFELD (GSCOP, GRENOBLE, FRANCE)

Binary pattern of length greater than 14 are abelian-2-avoidable.

Two words u and v are abelian equivalent if they are permutations of each other (abc and bac are abelian equivalent). If w is a word and $P = P_1 \dots P_n$ a pattern (a word over another alphabet, the P_i being the letters of P), we say that w contains an abelian occurrence of P if there exist non-empty words w_1, \dots, w_n such that $w_1 \dots w_n$ is a factor of w and for all i and j , $P_i = P_j$ implies that w_i and w_j are abelian equivalent. A pattern is avoidable if there is an infinite word that does not contain it. In 1992 Keränen gave a positive answer to a question from Erdős from 1961: there is an infinite word over 4 letters that avoids the pattern AA in the abelian sense. More recently, it was shown that every binary pattern of length greater than 118 is avoidable in the abelian sense. Here we improve the bound from 118 to 14 with a computer-assisted proof.

MASSIMILIANO ROSSI (UNIV. VERONA, ITALY)

Bubble-Flip – A New Generation Algorithm for Prefix Normal Words.

We present a new recursive generation algorithm for prefix normal words. These are binary strings with the property that no substring has more 1s than the prefix of the same length. The new algorithm uses two operations on binary strings, which exploit certain properties of prefix normal words in a smart way. We introduce infinite prefix normal words and show that one of the operations used by the algorithm, if applied repeatedly to extend the string, produces an ultimately periodic infinite word, which is prefix normal and whose period's length and density we can predict from the original word. Note: accepted at LATA 2018.

ANDREW RYZHIKOV (UNIV. PARIS EST, FRANCE)

Regular subsequences in finite words. A 17-year-old conjecture of Lyngso and Pedersen states that each binary circular word has a linear anti-palindromic subsequence of length at least $2/3$ of the length of the whole word. I will present several more conjectures of this spirit, obtained by exhaustive computer search. A similar conjecture can be formulated for palindromes instead of anti-palindromes. Stronger bounds are conjectured to hold for words without k consecutive equal letters, which relates the statements to the topic of complexity of words. I will also talk about palindromic subsequences in words (both circular and linear) over an alphabet of more than two letters, and give proofs of some weaker results, supporting my beliefs in the above-mentioned conjectures.

CAIUS WOJCIK (ICJ, LYON)

**Monochromatic factorisations and periodicity - Combinatorics on words
VS. Ramsey theory.**

In 2006, Brown asked whether or not, given a non-periodic word, one could find a coloring of its factors such that it admits no monochromatic factorisation. We will give here an affirmative and optimal solution, by giving such a coloring with only two colors. This kind of result highlights the limits of Ramsey theory in the context of combinatorics on words. The two theories will fight an epic battle over the math-battlefield, but as Kurt would say, who needs action when you got words? We will present a new question, linking ultimate periodicity to the existence of suffixes having super-monochromatic factorisation. We may end by presenting the case of the Zimin word.