Independent systems of Word Equations in three unknowns and Polynomials

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Big open problems

Are there three independent equations over three unknowns with a common solution?

Open even replacing "three" with an arbitrary larger number.

How many solutions can an equation with constants in one variable have?

- Aleksi Saarela, Systems of word equations, polynomials and linear algebra: A new approach, European J. Combin. 2015

- ŠH, Jan Žemlička, Algebraic properties of word equations, Journal of Algebra 2015

- Dirk Nowotka, Aleksi Saarela, One-Variable Word Equations and Three-Variable Constant-Free Word Equations, DLT 2016

$$P : \mathbb{N}^* \to \mathbb{Q}(\alpha)$$
$$a_0 a_1 a_2 \cdots a_n \mapsto a_0 + a_1\alpha + a_2\alpha^2 + \cdots + a_n\alpha^n$$

$$\mathscr{V}_E = (V_{E,X}, V_{E,Y}, V_{E,Z}) \in \mathbb{Q}(X, Y, Z)^3$$

$$\Omega_{L(\varphi)} : \mathbb{Q}(X, Y, Z) \to \mathbb{Q}(\alpha)$$
$$X \mapsto \alpha^{|\varphi(X)|} \quad Y \mapsto \alpha^{|\varphi(Y)|} \quad Z \mapsto \alpha^{|\varphi(Z)|}$$

$$\mathscr{P}(\varphi) = (P(\varphi(X)), P(\varphi(Y)), P(\varphi(Z))) \in \mathbb{Q}(\alpha)^3$$

$$\Omega_{L(\varphi)}(\mathscr{V}_E) \cdot \mathscr{P}(\varphi) = 0\,.$$